for critical pressure is

$$p_{cr} = \frac{4E}{[12(1-\mu^2)]^{1/2}}\left(\frac{t_s}{R}\right)^2\left\{\frac{\left[\dfrac{1+(A/bt_s)}{1+(1-\mu^2)\,A/bt_s}+\dfrac{A}{bt_s}\right]\left[1+\dfrac{12(1-\mu^2)I}{bt_s^3}+\dfrac{12(1-\mu^2)(d/t_s)^2}{1-\mu^2+(bt_s/A)}\right]}{2+(1+\mu)\,A/bt_s}\right\}^{1/2}$$

where I is the actual moment of inertia of the stiffener, and d is the distance from the skin midplane to the stiffener centroid. Equation (19) of the subject paper was used for E/G_3, and D_3/D was assumed negligible relative to unity.

The important differences between this formula and Professor Buchert's are the added term in the denominator necessary to lower the critical pressure to that for asymmetric buckling, and the larger numerical coefficient $4[12(1-\mu^2)]^{-1/2}$, which is the result of using classical theory. Perhaps the experimental results referred to in the previous comment should be correlated to this formula to provide a reduction factor and, if necessary, referred to a more appropriate formula. That is, a paper by Crawford[1] contains a further linear-theory correction to the previous formula that accounts for the asymmetric section geometry of this class of stiffening. It would be even more appropriate to use those results to establish a proper correlation factor between linear theory and experiment. The correction is made by simply adding to the previous equation the term

$$\frac{2EAd}{R^2b}\left\{\frac{1-(2\mu)/[1+(1-\mu^2)\,A/bt_s]}{2+(1+\mu)\,A/bt_s}\right\}$$

for stiffening on the convex surface. The term is subtracted for stiffening on the concave surface.

Equation (10) of the subject paper reduces to the following formula for critical pressure for local instability when the stiffeners provide nodes but no torsional restraint:

$$p_{cr} = \frac{3.62\,Et_s^3}{Rb_s^2}\left[1+\frac{3\,(1-\mu^2)}{\pi^4}\left(\frac{b}{R}\right)^4\left(\frac{R}{t_s}\right)^2\right]$$

For geometric proportions in the vicinity of their optimum value, the second term in this formula for local instability is less than 1% when $p/E < 10$. Equations (16) and (30), which are equal to the previous equation when its second term is neglected, are therefore appropriate as recommended in the paper when the stiffener's torsional stiffness is neglected for conservatism.

References

[1] Crawford, R. F., "Effects of asymmetric stiffening on buckling of shells," AIAA Preprint 65-371 (July 26–29, 1965).

Comments on "In-Plane Vibration of Spinning Disks"

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There are three aspects of the subject note[1] to be commented upon. First, since none of the four references cited by Huston deal with the in-plane vibrations of a spinning disk, the reader may have been left with the impression that no previous work had been done on this problem. The fact is that at least six earlier papers, Refs. 2–7, have dealt with the more complicated problem of the symmetric (and, in some cases, unsymmetric) in-plane vibrations of a spinning elastic disk attached to a hub of nonzero radius which includes, as a limiting case, the freely spinning disk.

Second, Eqs. (22) and (23) of Ref. 1 are of questionable physical significance, since the relative order of magnitude of the coriolis effects $\zeta R^2\Omega^2/E$ is of the same order of magnitude as the static strain produced in the disk by the centrifugal loading, and Huston's governing linear equations (1) and (2) already neglect terms of this order.

Third, in this writer's opinion, a satisfactory discussion of the effects of rotation on the frequencies of vibration of a spinning disk is yet to be given. For example, depending on whether the seemingly negligible terms $-\Omega^2 u$ and $-\Omega^2 v$ are added or not to the right-hand sides of Eqs. (1) and (2) of Ref. 1, one finds that, as the rotational frequency Ω increases, the lowest torsional mode of a disk attached to a central hub becomes unstable[4,7] or remains stable,[2,3,6] respectively. Thus, were Huston's equations to be applied to this problem, his argument for neglecting these terms as simply being small[8] would have to be modified. It appears that any complete treatment of the problem should include the effects of the initial, static stresses on the vibration. Some discussion on this point may be found in Ref. 6.

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References

[1] Huston, R. L., "In-plane vibration of spinning disks," AIAA J. **3**, 1519–1520 (1965).

[2] Grammel, R., "Drillungs-und Dehungsschwingungen unlaufender Schieben," Ing-Arch. **6**, 256–265 (1935).

[3] Biezeno, C. B. and Grammel, R., *Engineering Eynamics: Steam Turbines*, translated by E. F. Winter and H. A. Havemann (Blackie and Sons, Ltd., London, 1954), Vol. III.

[4] Yamada, K., "Vibration of turbine disc in its plane," *Proceedings of the 2nd Japan National Congress Applied Mechanics* (National Committee on Theoretical and Applied Mechanics, Science Council of Japan, Tokyo, 1953), pp. 343–347.

[5] Singh, B. R. and Nandeeswaraiya, N. S., "Vibration analysis of turbine disc in its plane," J. Sci. Eng. Res. (India) **1**, 157–160 (1957).

[6] Simmonds, J. G., "The in-plane vibration of a flat spinning disk," NASA TN D-521 (December 1962).

[7] Bhuta, P. G. and Jones, J. P., "Symmetric planar vibrations of a rotating disk," J. Acoust. Soc. Am. **35**, 982–989 (1963).

[8] Huston, R. L., "Wave propagation in rotating elastic media," AIAA J. 2, 575–576 (1964).

Reply by Author to J. G. Simmonds

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Perhaps it is convenient to reply to these statements in order. First, I extend my apologies to J. G. Simmonds and any others I may have overlooked in the references. I chose primarily those works that I needed for the development of the subject note,[1] the purpose being to investigate the effect of the Coriolis acceleration. In much of the work done by others,[2,3] this effect apparently is discarded as being negligible.

Second, a mathematical model can only be expected to predict physical phenomena in view of the assumptions used in

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its construction. In the construction of the mathematical model of the subject note, the assumptions[4] were intended to be mutually consistent with those of linear elasticity theory. Regarding this, an order of magnitude analysis[5] shows that terms of the type $\Omega^2 u$ are of the same order as certain nonlinear terms, which are, of course, neglected in the linear theory. Hence, terms of the type $\Omega^2 u$, even though they are linear, were regarded as small and neglected. The centrifugal forces then produce a static deformation about which the vibratory motion takes place. Because the equations are linear, and only small deformation is considered, this static deformation may be a accounted for by superposing a particular solution of Eq. (3) of Ref. 4.

Third, a discussion that would consider the effects of terms of the type $\Omega^2 u$ and the effects of the initial static stress on the vibratory motion should, in order to be consistent, also include the notions of nonlinear strain, nonlinear stress-strain equations, initial and deformed coordinates, etc. It was not my intention to consider these in the subject note.

References

[1] Huston, R. L., "In-plane vibration of spinning disks," AIAA J. **3,** 1519–1520 (1965).

[2] Simmonds, J. G., "The in-plane vibration of a flat spinning disk," NASA TN D521 (December 1962).

[3] Biezens, C. B. and Grammel, R., *Steam Turbines: Engineering Dynamics,* transl. by E. F. Winter and H. A. Havemann (Blackie and Sons, Ltd., London, England, 1954), Vol. III.

[4] Huston, R. L., "Wave propagation in rotating elastic media," AIAA J. 2, 575–576 (1964).

[5] Huston, R. L., "On the vibrations of elastic bodies in the presence of rigid body motion," Ph.D. dissertation, Univ. of Pennsylvania, Philadelphia, Pa. (1962).

Comments on "Bow Shock Shape about a Spherical Nose"

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In a recent Technical Note, Berman[1] has presented a simplified method to predict the bow shock-wave profile about a spherical nose for Mach numbers greater than five. The results of the analysis are contained in two expressions, one of which is more accurate and utilizes the factors of radius ratio, eccentricity, and shock detachment distance as functions of density ratio across a normal shock.

It is interesting to note that a similar correlation concept of the bow shock profile for spheres utilizing the density ratio across a normal shock was employed in Ref. 2 in a study of hypersonic blunt-body similitude. Gregorek and the present writer[2] began with the general form of the equation for the spherical shock as obtained from blast wave analogy, i.e.,

$$r_s/d_N = A(x/d_N)^n \tag{1}$$

where the coordinate system is identical to Ref. 1 with the exception that the origin is at the shock apex. It has been shown by previous results[3] that the usual values associated with A and n, arrived at by blast wave theory, are inadequate in the prediction of the shock-wave profiles about spheres for $x/d_N < 4$.

The approach used in Ref. 2 was to obtain photographs of the bow shock profiles about spheres by means of a glow discharge apparatus installed in the 4-in. continuous, freejet, hypersonic wind tunnel of The Ohio State University. These photographs were projected on a grid, and the nondimensionalized coordinates r_s/d_N and x/d_N were determined. When the data thus obtained are displayed on a logarithmic scale, the profiles may be observed to be nearly linear and may be expressed in the form of Eq. (1).

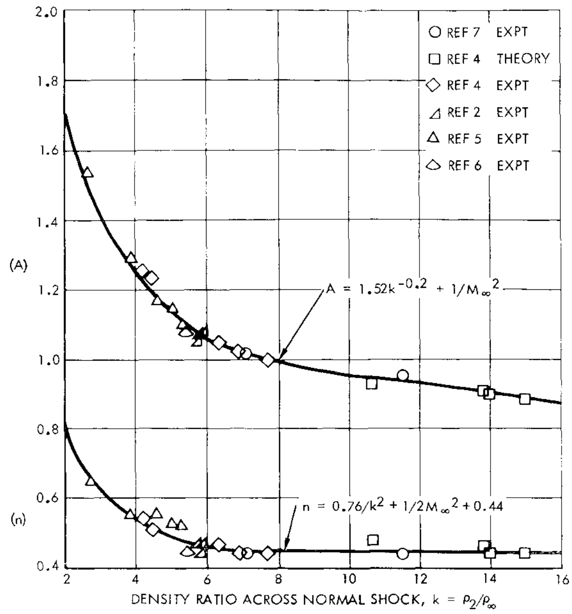

Fig. 1 Dependence of shock-wave constants on density ratio across normal shock.

Employing the shock-wave constants obtained for spheres from this study[2] and from sphere shock profiles presented in investigations by Seiff and Whiting,[4] Baer,[5] Love,[6] and Lobb,[7] values for A and n were plotted against density ratio as shown in Fig. 1. From these results, an empirical correlation for A and n based on density ratio and Mach number was formulated and took the form shown in Fig. 1. The experimental values of A are well represented by its empirical expression, with the maximum deviation of any of these points being less than 2%. Values of the exponent n show more

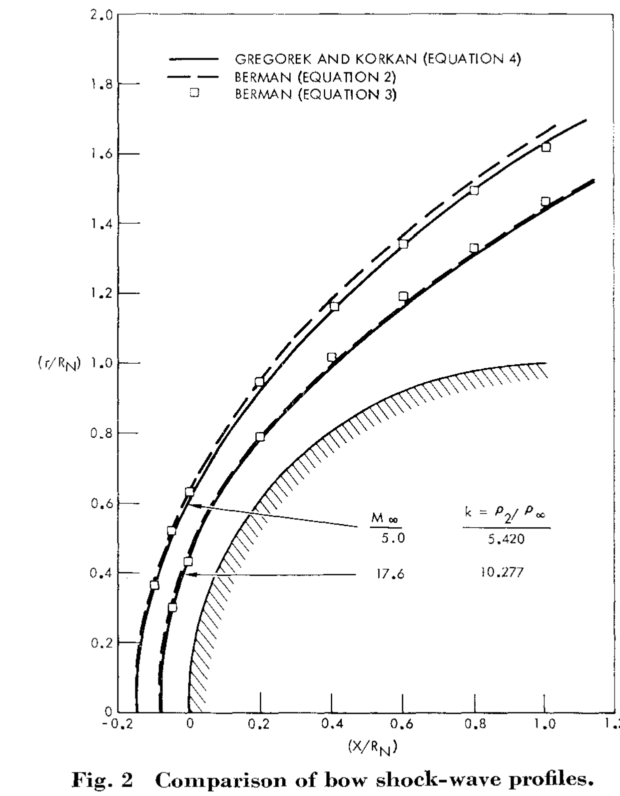

Fig. 2 Comparison of bow shock-wave profiles.

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